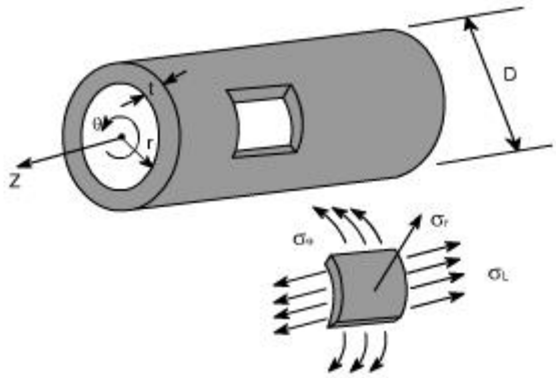
Pressurized thin walled cylinder:

Preamble : Pressure vessels are exceedingly important in industry. Normally two types of pressure vessel are used in common practice such as cylindrical pressure vessel and spherical pressure vessel.

In the analysis of this walled cylinders subjected to internal pressures it is assumed that the radial plans remains radial and the wall thickness dose not change due to internal pressure. Although the internal pressure acting on the wall causes a local compressive stresses (equal to pressure) but its value is neglibly small as compared to other stresses & hence the sate of stress of an element of a thin walled pressure is considered a biaxial one.

Further in the analysis of them walled cylinders, the weight of the fluid is considered neglible.

Let us consider a long cylinder of circular cross - section with an internal radius of R_2 and a constant wall thickness't' as showing fig.



This cylinder is subjected to a difference of hydrostatic pressure of 'p' between its inner and outer surfaces. In many cases, 'p' between gage pressure within the cylinder, taking outside pressure to be ambient.

By thin walled cylinder we mean that the thickness't' is very much smaller than the radius R_i and we may quantify this by stating than the ratio t / R_i of thickness of radius should be less than 0.1.

An appropriate co-ordinate system to be used to describe such a system is the cylindrical polar one r, q , z shown, where z axis lies along the axis of the cylinder, r is radial to it and q is the angular co-ordinate about the axis.

The small piece of the cylinder wall is shown in isolation, and stresses in respective direction have also been shown.

Type of failure:

Such a component fails in since when subjected to an excessively high internal pressure. While it might fail by bursting along a path following the circumference of the cylinder. Under normal circumstance it fails by circumstances it fails by bursting along a path parallel to the axis. This suggests that the hoop stress is significantly higher than the axial stress.

In order to derive the expressions for various stresses we make following

Applications :

Liquid storage tanks and containers, water pipes, boilers, submarine hulls, and certain air plane components are common examples of thin walled cylinders and spheres, roof domes.

ANALYSIS : In order to analyse the thin walled cylinders, let us make the following assumptions :

- There are no shear stresses acting in the wall.
- The longitudinal and hoop stresses do not vary through the wall.
- Radial stresses s_r which acts normal to the curved plane of the isolated element are neglibly small as compared to other two stresses especially when $\left[\frac{t}{R_i} < \frac{1}{20}\right]$

The state of tress for an element of a thin walled pressure vessel is considered to be biaxial, although the internal pressure acting normal to the wall causes a local compressive stress equal to the internal pressure, Actually a state of tri-axial stress exists on the inside of the vessel. However, for then walled pressure vessel the third stress is much smaller than the other two stresses and for this reason in can be neglected.

Thin Cylinders Subjected to Internal Pressure:

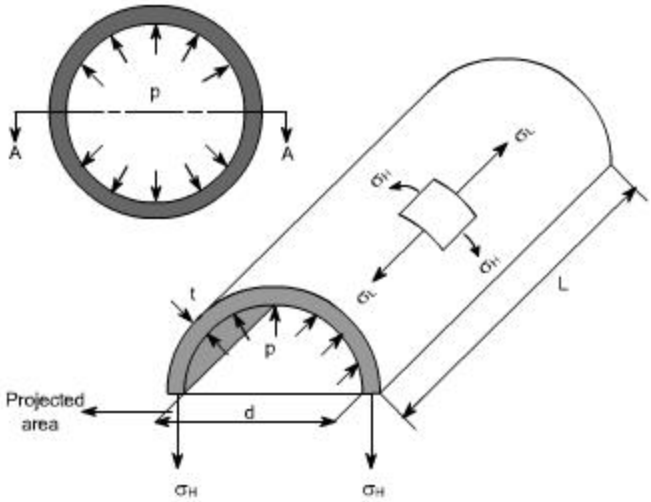
When a thin – walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylinder materials, namely

- Circumferential or hoop stress
- The radial stress
- Longitudinal stress

now let us define these stresses and determine the expressions for them

Hoop or circumferential stress:

This is the stress which is set up in resisting the bursting effect of the applied pressure and can be most conveniently treated by considering the equilibrium of the cylinder.



In the figure we have shown a one half of the cylinder. This cylinder is subjected to an internal pressure p.

i.e. p = internal pressure

d = inside diametre

L = Length of the cylinder

t = thickness of the wall

Total force on one half of the cylinder owing to the internal pressure 'p'

= p x Projected Area

= p x d x L

= p .d. L ------- (1)

The total resisting force owing to hoop stresses s_H set up in the cylinder walls

= 2 .s_H .L.t ---------(2)

Because s_H.L.t. is the force in the one wall of the half cylinder.

the equations (1) & (2) we get

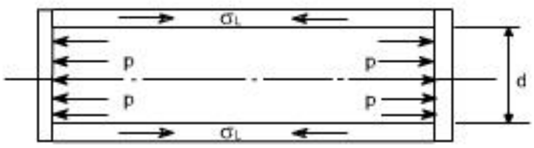
2 . s_H . L . t = p . d . L

s_H = (p . d) / 2t

Circumferential or hoop Stress (s_H) = (p .d)/ 2t

Longitudinal Stress:

Consider now again the same figure and the vessel could be considered to have closed ends and contains a fluid under a gage pressure p.Then the walls of the cylinder will have a longitudinal stress as well as a ciccumferential stress.



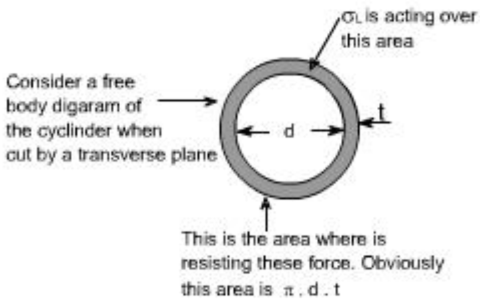
Total force on the end of the cylinder owing to internal pressure

= pressure x area

= p x p d^2 /4

Area of metal resisting this force = pd.t. (approximately)

because pd is the circumference and this is multiplied by the wall thickness



Hence the longitudnal stresses

$$\text{Set up} = \frac{\text{force}}{\text{area}} = \frac{[p \times \pi d^2/4]}{\pi dt}$$

$$= \frac{pd}{4t} \quad \text{or} \quad \sigma_L = \frac{pd}{4t}$$

or alternatively from equilibrium conditions

$$\sigma_L.(\pi dt) = p.\frac{\pi d^2}{4}$$

$$\text{Thus } \boxed{\sigma_L = \frac{pd}{4t}}$$

LAMES EQUATION FOR THICK CYLINDERS

When the material of the cylinder is brittle, such as Cast iron or Cast steel, Lame's equation is used to determine the wall thickness. It is based on the Maximum Principal stress theory, where maximum principal stress is equated to permissible stress of the material.

The three principal stresses at the inner surface of the cylinder are:

- Tangential or hoop stress σ_t
- Longitudinal stress σ^l_t
- Radial stress σ_r

Thick Cylinders

Lame's Equations:

The tangential stress in the cylinder wall at radius r

$$\sigma_t = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} + \frac{d_i^2 d_o^2 (p_i - p_o)}{4r^2 (d_o^2 - d_i^2)}$$
$$= a + \frac{b}{r^2}$$

The radial stress in the cylinder wall at radius r

$$\sigma_r = \frac{p_i d_i^2 - p_o d_o^2}{d_o^2 - d_i^2} - \frac{d_i^2 d_o^2 (p_i - p_o)}{4r^2 (d_o^2 - d_i^2)}$$
$$= a - \frac{b}{r^2}$$

Lame's equation for internal pressure:

The tangential stress at radius r, $\sigma_t = \frac{p_i d_i^2}{4r^2} \left(\frac{4r^2 + d_o^2}{d_o^2 - d_i^2} \right)$

The radial stress at radius r, $\sigma_r = \frac{p_i d_i^2}{4r^2} \left(\frac{4r^2 - d_o^2}{d_o^2 - d_i^2} \right)$

The maximum tangential stress at the inner surface, $\sigma_{t\max} = \frac{p_i (d_o^2 + d_i^2)}{d_o^2 - d_i^2}$

The maximum shear stress at the inner surface, $\tau_{\max} = \frac{p_i d_o^2}{d_o^2 - d_i^2}$

The cylinder wall thickness for brittle materials based on the maximum normal stress theory

$$t = \frac{d_i}{2} \left[\left(\frac{\sigma_t + p_i}{\sigma_t - p_i} \right)^{\frac{1}{2}} - 1 \right]$$

Clavarino's equation is applicable to cylinders with closed ends and made of ductile materials. Clavarino's equation is based on maximum strain theory.

The thickness of a thick cylinrer based on Clavarino's equation is given by

$$t = \frac{d_i}{2}\left[\left(\frac{\sigma_t' + (1-2\mu)p_i}{\sigma_t' - (1+\mu)p_i}\right)^{\frac{1}{2}} - 1\right]$$

Birnie's equation for open ended cylinders made of ductile materials is given by

$$t = \frac{d_i}{2}\left[\left(\frac{\sigma_t' + (1-\mu)p_i}{\sigma_t' - (1+\mu)p_i}\right)^{\frac{1}{2}} - 1\right]$$